

Limits and Derivatives

Question 1.

The expansion of $\log(1 - x)$ is

- (a) $x - x^2/2 + x^3/3 - \dots$
- (b) $x + x^2/2 + x^3/3 + \dots$
- (c) $-x + x^2/2 - x^3/3 + \dots$
- (d) $-x - x^2/2 - x^3/3 - \dots$

Answer: (d) $-x - x^2/2 - x^3/3 - \dots$

$\log(1 - x) = -x - x^2/2 - x^3/3 - \dots$

Question 2.

The value of $\lim_{x \rightarrow a} (a \times \sin x - x \times \sin a)/(ax^2 - xa^2)$ is

- (a) $= (a \times \cos a + \sin a)/a^2$
- (b) $= (a \times \cos a - \sin a)/a^2$
- (c) $= (a \times \cos a + \sin a)/a$
- (d) $= (a \times \cos a - \sin a)/a$

Answer: (b) $= (a \times \cos a - \sin a)/a^2$

Given,

$$\lim_{x \rightarrow a} (a \times \sin x - x \times \sin a)/(ax^2 - xa^2)$$

When we put $x = a$ in the expression, we get $0/0$ form.

Now apply L. Hospital rule, we get

$$\lim_{x \rightarrow a} (a \times \cos x - \sin a)/(2ax - a^2)$$

$$= (a \times \cos a - \sin a)/(2a \times a - a^2)$$

$$= (a \times \cos a - \sin a)/(2a^2 - a^2)$$

$$= (a \times \cos a - \sin a)/a^2$$

$$\text{So, } \lim_{x \rightarrow a} (a \times \sin x - x \times \sin a)/(ax^2 - xa^2) = (a \times \cos a - \sin a)/a^2$$

Question 3.

$\lim_{x \rightarrow -1} [1 + x + x^2 + \dots + x^{10}]$ is

- (a) 0
- (b) 1
- (c) -1
- (d) 2

Answer: (b) 1

$$\begin{aligned}\text{Given, } \lim_{x \rightarrow -1} [1 + x + x^2 + \dots + x^{10}] \\&= 1 + (-1) + (-1)^2 + \dots + (-1)^{10} \\&= 1 - 1 + 1 - \dots + 1 \\&= 1\end{aligned}$$

Question 4.

The value of the limit $\lim_{x \rightarrow 0} \{\log(1 + ax)\}/x$ is

- (a) 0
- (b) 1
- (c) a
- (d) 1/a

Answer: (c) a

$$\begin{aligned}\text{Given, } \lim_{x \rightarrow 0} \{\log(1 + ax)\}/x \\&= \lim_{x \rightarrow 0} \{ax - (ax)^2/2 + (ax)^3/3 - (ax)^4/4 + \dots\}/x \\&= \lim_{x \rightarrow 0} \{ax - a^2 x^2/2 + a^3 x^3/3 - a^4 x^4/4 + \dots\}/x \\&= \lim_{x \rightarrow 0} \{a - a^2 x/2 + a^3 x^2/3 - a^4 x^3/4 + \dots\} \\&= a - 0 \\&= a\end{aligned}$$

Question 5.

The value of the limit $\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x}$ is

- (a) 1
- (b) e
- (c) $e^{1/2}$
- (d) $e^{-1/2}$

Answer: (d) $e^{-1/2}$

$$\begin{aligned}\text{Given, } \lim_{x \rightarrow 0} (\cos x)^{\cot^2 x} \\&= \lim_{x \rightarrow 0} (1 + \cos x - 1)^{\cot^2 x} \\&= e^{\lim_{x \rightarrow 0} (\cos x - 1) \times \cot^2 x}\end{aligned}$$

$$= e^{\lim_{x \rightarrow 0} (\cos x - 1)/\tan^2 x}$$

$$= e^{-1/2}$$

Question 6.

Then value of $\lim_{x \rightarrow 1} (1 + \log x - x)/(1 - 2x + x^2)$ is

- (a) 0
- (b) 1
- (c) 1/2
- (d) -1/2

Answer: (d) -1/2

$$\begin{aligned} \text{Given, } & \lim_{x \rightarrow 1} (1 + \log x - x)/(1 - 2x + x^2) \\ &= \lim_{x \rightarrow 1} (1/x - 1)/(-2 + 2x) \quad \{\text{Using L. Hospital Rule}\} \\ &= \lim_{x \rightarrow 1} (1 - x)/\{2x(x - 1)\} \\ &= \lim_{x \rightarrow 1} (-1/2x) \\ &= -1/2 \end{aligned}$$

Question 7.

The value of $\lim_{y \rightarrow 0} \{(x + y) \times \sec(x + y) - x \times \sec x\}/y$ is

- (a) $x \times \tan x \times \sec x$
- (b) $x \times \tan x \times \sec x + x \times \sec x$
- (c) $\tan x \times \sec x + \sec x$
- (d) $x \times \tan x \times \sec x + \sec x$

Answer: (d) $x \times \tan x \times \sec x + \sec x$

$$\begin{aligned} \text{Given, } & \lim_{y \rightarrow 0} \{(x + y) \times \sec(x + y) - x \times \sec x\}/y \\ &= \lim_{y \rightarrow 0} \{x \sec(x + y) + y \sec(x + y) - x \times \sec x\}/y \\ &= \lim_{y \rightarrow 0} [x\{\sec(x + y) - \sec x\} + y \sec(x + y)]/y \\ &= \lim_{y \rightarrow 0} x\{\sec(x + y) - \sec x\}/y + \lim_{y \rightarrow 0} \{y \sec(x + y)\}/y \\ &= \lim_{y \rightarrow 0} x\{1/\cos(x + y) - 1/\cos x\}/y + \lim_{y \rightarrow 0} \{y \sec(x + y)\}/y \\ &= \lim_{y \rightarrow 0} [\{\cos x - \cos(x + y)\} \times x/\{y \times \cos(x + y) \times \cos x\}] + \lim_{y \rightarrow 0} \{y \sec(x + y)\}/y \\ &= \lim_{y \rightarrow 0} [\{2\sin(x + y/2) \times \sin(y/2)\} \times 2x/\{2y \times \cos(x + y) \times \cos x\}] + \lim_{y \rightarrow 0} \{y \sec(x + y)\}/y \\ &= \lim_{y \rightarrow 0} \{\sin(x + y/2) \times \lim_{y \rightarrow 0} \{\sin(y/2)/(2y/2)\} \times \lim_{y \rightarrow 0} \{x/\{y \times \cos(x + y) \times \cos x\}\} + \sec x \\ &= \sin x \times 1 \times x/\cos^2 x + \sec x \\ &= x \times \tan x \times \sec x + \sec x \\ \text{So, } & \lim_{y \rightarrow 0} \{(x + y) \times \sec(x + y) - x \times \sec x\}/y = x \times \tan x \times \sec x + \sec x \end{aligned}$$

Question 8.

$\lim_{x \rightarrow 0} (e^{x^2} - \cos x)/x^2$ is equals to

- (a) 0
- (b) 1
- (c) $2/3$
- (d) $3/2$

Answer: (d) $3/2$

Given, $\lim_{x \rightarrow 0} (e^{x^2} - \cos x)/x^2$

$$= \lim_{x \rightarrow 0} (e^{x^2} - \cos x - 1 + 1)/x^2$$

$$= \lim_{x \rightarrow 0} \{(e^{x^2} - 1)/x^2 + (1 - \cos x)\}/x^2$$

$$= \lim_{x \rightarrow 0} \{(e^{x^2} - 1)/x^2 + \lim_{x \rightarrow 0} (1 - \cos x)\}/x^2$$

$$= 1 + 1/2$$

$$= (2 + 1)/2$$

$$= 3/2$$

Question 9.

The expansion of a^x is

(a) $a^x = 1 + x/1! \times (\log a) + x^2/2! \times (\log a)^2 + x^3/3! \times (\log a)^3 + \dots$

(b) $a^x = 1 - x/1! \times (\log a) + x^2/2! \times (\log a)^2 - x^3/3! \times (\log a)^3 + \dots$

(c) $a^x = 1 + x/1 \times (\log a) + x^2/2 \times (\log a)^2 + x^3/3 \times (\log a)^3 + \dots$

(d) $a^x = 1 - x/1 \times (\log a) + x^2/2 \times (\log a)^2 - x^3/3 \times (\log a)^3 + \dots$

Answer: (a) $a^x = 1 + x/1! \times (\log a) + x^2/2! \times (\log a)^2 + x^3/3! \times (\log a)^3 + \dots$

$$a^x = 1 + x/1! \times (\log a) + x^2/2! \times (\log a)^2 + x^3/3! \times (\log a)^3 + \dots$$

Question 10.

The value of the limit $\lim_{n \rightarrow 0} (1 + an)^{b/n}$ is

(a) e^a

(b) e^b

(c) e^{ab}

(d) $e^{a/b}$

Answer: (c) e^{ab}

Given, $\lim_{n \rightarrow 0} (1 + an)^{b/n}$

$$= e^{\lim_{n \rightarrow 0} (an \times b/n)}$$

$$= e^{\lim_{n \rightarrow 0}(ab)}$$

$$= e^{ab}$$

Question 11.

The value of $\lim_{x \rightarrow 0} \cos x / (1 + \sin x)$ is

- (a) 0
- (b) -1
- (c) 1
- (d) None of these

Answer: (c) 1

Given, $\lim_{x \rightarrow 0} \cos x / (1 + \sin x)$

$$= \cos 0 / (1 + \sin 0)$$

$$= 1 / (1 + 0)$$

$$= 1/1$$

$$= 1$$

Question 12.

$\lim_{x \rightarrow \pi/4} \tan 2x \times \tan(\pi/4 - x)$ is

- (a) 0
- (b) 1
- (c) 1/2
- (d) 3/2

Answer: (c) 1/2

Given, $\lim_{x \rightarrow \pi/4} \tan 2x \times \tan(\pi/4 - x)$

$$= \lim_{h \rightarrow 0} \tan 2(\pi/4 - x) \times \tan(-h)$$

$$= \lim_{h \rightarrow 0} -\cot 2h / (-\cot h)$$

$$= \lim_{h \rightarrow 0} \tan h / \tan 2h$$

$$= (1/2) \times \lim_{h \rightarrow 0} (\tan h / h) / (2h / \tan 2h)$$

$$= (1/2) \times \{ \lim_{h \rightarrow 0} (\tan h / h) \} / \{ \lim_{h \rightarrow 0} (2h / \tan 2h) \}$$

$$= (1/2) \times 1$$

$$= 1/2$$

Question 13.

$\lim_{x \rightarrow 2} (x^3 - 6x^2 + 11x - 6) / (x^2 - 6x + 8) =$

- (a) 0
- (b) 1

- (c) $1/2$
(d) Limit does not exist

Answer: (c) $1/2$

When $x = 2$, the expression

$(x^3 - 6x^2 + 11x - 6)/(x^2 - 6x + 8)$ assumes the form $0/0$

Now,

$$\begin{aligned}\lim_{x \rightarrow 2} (x^3 - 6x^2 + 11x - 6)/(x^2 - 6x + 8) &= \lim_{x \rightarrow 2} \{(x-1) \times (x-2) \times (x-3)\} / \{(x-2) \times (x-4)\} \\ &= \lim_{x \rightarrow 2} \{(x-1) \times (x-3)\} / (x-4) \\ &= \{(2-1) \times (2-3)\} / (2-4) \\ &= 1/2\end{aligned}$$

Question 14.

The value of the limit $\lim_{x \rightarrow 2} (x-2)/\sqrt{2-x}$ is

- (a) 0
(b) 1
(c) -1
(d) 2

Answer: (a) 0

Given, $\lim_{x \rightarrow 2} (x-2)/\sqrt{2-x}$

$$\begin{aligned}&= \lim_{x \rightarrow 2} -(2-x)/\sqrt{2-x} \\ &= \lim_{x \rightarrow 2} -\{\sqrt{2-x} \times \sqrt{2-x}\} / \sqrt{2-x} \\ &= \lim_{x \rightarrow 2} -\sqrt{2-x} \\ &= -\sqrt{2-2} \\ &= 0\end{aligned}$$

Question 15.

The derivative of the function $f(x) = 3x^3 - 2x^2 + 5x - 1$ at $x = -1$ is

- (a) 0
(b) 1
(c) -18
(d) 18

Answer: (d) 18

Given, function $f(x) = 3x^3 - 2x^2 + 5x - 1$

Differentiate w.r.t. x , we get

$$\begin{aligned}df(x)/dx &= 3 \times 3 \times x^2 - 2 \times 2 \times x + 5 \\ \Rightarrow df(x)/dx &= 9x^2 - 4x + 5\end{aligned}$$

$$\Rightarrow \{df(x)/dx\}_{x=-1} = 9 \times (-1)^2 - 4 \times (-1) + 5$$

$$\Rightarrow \{df(x)/dx\}_{x=-1} = 9 + 4 + 5$$

$$\Rightarrow \{df(x)/dx\}_{x=-1} = 18$$

Question 16.

$\lim_{x \rightarrow 0} \sin^2(x/3)/x^2$ is equals to

- (a) 1/2
- (b) 1/3
- (c) 1/4
- (d) 1/9

Answer: (d) 1/9

Given, $\lim_{x \rightarrow 0} \sin^2(x/3)/x^2$

$$= \lim_{x \rightarrow 0} [\sin^2(x/3)/(x/3)^2 \times \{(x/3)^2/x^2\}]$$

$$= \lim_{x \rightarrow 0} [\{\sin(x/3)/(x/3)\}^2 \times \{(x^2/9)/x^2\}]$$

$$= 1 \times 1/9$$

$$= 1/9$$

Question 17.

The expansion of a^x is

- (a) $a^x = 1 + x/1! \times (\log a) + x^2/2! \times (\log a)^2 + x^3/3! \times (\log a)^3 + \dots$
- (b) $a^x = 1 - x/1! \times (\log a) + x^2/2! \times (\log a)^2 - x^3/3! \times (\log a)^3 + \dots$
- (c) $a^x = 1 + x/1 \times (\log a) + x^2/2 \times (\log a)^2 + x^3/3 \times (\log a)^3 + \dots$
- (d) $a^x = 1 - x/1 \times (\log a) + x^2/2 \times (\log a)^2 - x^3/3 \times (\log a)^3 + \dots$

Answer: (a) $a^x = 1 + x/1! \times (\log a) + x^2/2! \times (\log a)^2 + x^3/3! \times (\log a)^3 + \dots$

$$ax = 1 + x/1! \times (\log a) + x^2/2! \times (\log a)^2 + x^3/3! \times (\log a)^3 + \dots$$

Question 18.

Differentiation of $\cos \sqrt{x}$ with respect to x is

- (a) $\sin x / 2\sqrt{x}$
- (b) $-\sin x / 2\sqrt{x}$
- (c) $\sin \sqrt{x} / 2\sqrt{x}$
- (d) $-\sin \sqrt{x} / 2\sqrt{x}$

Answer: (d) $-\sin \sqrt{x} / 2\sqrt{x}$

Let $y = \cos \sqrt{x}$

Put $u = \sqrt{x}$

$$\begin{aligned} du/dx &= 1/2\sqrt{x} \\ \text{Now, } y &= \cos u \\ dy/du &= -\sin u \\ dy/dx &= (dy/du) \times (du/dx) \\ &= -\sin u \times (1/2\sqrt{x}) \\ &= -\sin \sqrt{x} / 2\sqrt{x} \end{aligned}$$

Question 19.

Differentiation of $\log(\sin x)$ is

- (a) cosec x
- (b) cot x
- (c) sin x
- (d) cos x

Answer: (b) cot x

Let $y = \log(\sin x)$

Again let $u = \sin x$

$$du/dx = \cos x$$

Now, $y = \log u$

$$dy/du = 1/u = 1/\sin x$$

$$\text{Now, } dy/dx = (dy/du) \times (du/dx)$$

$$\Rightarrow dy/dx = (1/\sin x) \times \cos x$$

$$\Rightarrow dy/dx = \cos x / \sin x$$

$$\Rightarrow dy/dx = \cot x$$

Question 20.

$\lim_{x \rightarrow \infty} \{(x+5)/(x+1)\}^x$ equals

- (a) e^2
- (b) e^4
- (c) e^6
- (d) e^8

Answer: (c) e^6

Given, $\lim_{x \rightarrow \infty} \{(x+5)/(x+1)\}^x$

$$= \lim_{x \rightarrow \infty} \{1 + 6/(x+1)\}^x$$

$$= e^{\lim_{x \rightarrow \infty} 6x/(x+1)}$$

$$= e^{\lim_{x \rightarrow \infty} 6/(1+1/x)}$$

$$= e^{6/(1+1/\infty)}$$

$$= e^{6/(1+0)}$$

$$= e^6$$

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